An Alternative Ionospheric Correction Algorithm for Satellite-Based Augmentation Systems in Low-Latitude Region

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1. Abstract

The ionospheric correction algorithms have been characterized extensively for the mid-latitude region of the ionosphere where benign conditions usually exist. The United States Federal Aviation Administration s (FAA) Wide Area Augmentation System (WAAS) for civil aircraft navigation is focused primarily on the Conterminous United States (CONUS). Other Satellite-Based Augmentation Systems (SBAS) include the European Geostationary Navigation Overlay Service (EGNOS) and the Japanese Global Navigation Satellite System (MSAS). Researchers are facing a more formidable challenge in addressing the ionospheric impact on navigation using SBAS in other parts of the world such as the South American region or India. At equatorial latitudes, geophysical conditions lead to the so-called Appleton-Hartree (equatorial) anomaly phenomenon, which results in significantly larger ionospheric range delays and range delay spatial gradients than are observed in the CONUS or European sectors.

In this paper, we use data from the South American region to perform a quantitative assessment of WAAS-type ionospheric correction algorithms in this region. For the study, we accessed a world-wide network of 400+ dual-frequency GPS receivers. The network includes: 1) the Continuously Operating Reference Sites (CORS) in the United States; 2) stations in and near South America as part of the Brazilian Network of Continuous Monitoring of GPS (RBMC), operated by the Brazilian Institute of Geography and Statistics (IGBE); and (3) sites included in the International GPS Service (IGS) global network. Data sets have been selected to include both quiet and storm days. To provide ground-truth and calibrate GPS receiver and transmitter inter-frequency biases, we processed the GPS data using Global Ionospheric Mapping (GIM) software developed at the Jet Propulsion Laboratory to compute calibrated high resolution observations of ionospheric total electron content (TEC).

In this study we investigated the performance of quadratic fit models to augment or replace the WAAS algorithm in Brazil. We found that for quiet days, the WAAS planar fit residuals are less than 13 meters with 1.42 meter root-mean square (RMS). When using a quadratic approach, we

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obtained residuals less than 9 meters with 1.29 meter RMS. After investigating a number of storm and quiet days, we concluded that the quadratic approach resulted on average in a 20 percent improvement in accuracy over the planar fit approach.

2. Introduction

The Wide-Area Augmentation System (WAAS) developed for the Conterminous United States (CONUS) is only one of the several Space-Based Augmentation Systems (SBAS) under consideration worldwide. Other SBAS developments are under way in Europe, Japan, India and Brazil.

Relatively benign ionospheric conditions in the mid-latitude CONUS region are compatible with accurate ionospheric range corrections for WAAS. Providing ionospheric corrections for Brazil is significantly more challenging, since ionospheric range delays and range delay spatial gradients are among the largest in the world even in the absence of ionospheric storms (during infrequent ionospheric storms, even mid-latitude regions present challenging conditions). In summary, the ionosphere in the Brazilian sector shows significantly different behavior from that of the mid-latitude sector.

The ionosphere has been extensively studied to support WAAS at the CONUS sector. The published literature discussing ionospheric corrections for WAAS in the CONUS is extensive; see e.g. Enge et al., [1996], WAAS MOPS [1999], Walter et al., [2000] and Sparks et al., [2002]. Various alternative ionospheric correction algorithms have been presented by e.g., Hansen et al., [1997], Sparks et al., [2000] and Blanch et al., [2002]. A potential application of WAAS algorithms to Brazil has recently been investigated by Klobuchar et al., [2002] using simulated data. The temporal and spatial variability of the low-latitude ionosphere was studied in the context of ionospheric storms by Dehel and Corbelli [2002] and Fedrizzi et al., [2001] using a network of dual-frequency GPS receivers in Brazil. Investigating the possible application of the current WAAS algorithm using actual GPS data is the natural progression of the previous studies and therefore the main focus of this paper.

In Komjathy et al., [2002b], we assessed the WAAS s planar fit algorithm in the equatorial region where the spatial gradients and the absolute slant TEC are known to be the highest in the world. We found that in Brazil the dominant error source for the WAAS planar fit algorithm is the inherent spatial variability of the equatorial ionosphere with ionospheric slant range delay residuals as high as 15 meters and root-mean square (RMS) residuals for the quiet day of 1.9 meters. This compares to a maximum residual of 2 meters in CONUS, and 0.5 meter RMS. We revealed that ionospheric gradients in Brazil are on average at the level of 2 meters over 100 km. Contrary to results obtained for CONUS, we discovered that a major ionospheric storm (March 31, 2001) had a small impact on the planar fit residuals in Brazil.

In this research, we first review the estimation method used to solve for inter-frequency biases (nuisance parameters) in the GPS satellites and receivers, using a global network of 230 GPS sites in order to provide ground truth data for the analysis. Subsequently, we describe the WAAS planar fit algorithm used to estimate the vertical ionospheric range delay at fixed latitude and longitude locations known as ionospheric grid points (IGPs). This will be followed by examining the application of using a higher order, quadratic fit to augment or replace the WAAS algorithm in Brazil. We compare ionospheric range residuals using planar and quadratic fits in the Brazilian sector.

3. GIM bias estimaton strategy

To provide ground-truth, we used the Global Ionospheric Mapping (GIM) software developed at the Jet Propulsion Laboratory [Mannucci et al., 1998] to compute high precision slant ionospheric delay by removing the satellite and receiver differential biases from the ionospheric observables, generated from carrier-phase data adjusted to match the ionospheric delay based on dual-frequency pseudoranges. The estimation of the satellite and receiver biases is described here briefly.

Ionospheric measurements from a GPS receiver can be modeled with the well-known single-shell ionospheric model using the following observation equation [see e.g. Mannucci et al., 1999 and Komjathy et al., 2002a]:

$$\mathrm{TEC} = \mathrm{M(h,E)} \sum_{\mathrm{i}} \mathrm{C_i B_i (lat,lon)} + \mathrm{b_r} + \mathrm{b_s} \, , \qquad (1)$$

where

TEC	is the slant Total Electron Content measured by the linear combination of the GPS dual-frequency carrier phase and pseudorange ionospheric observables, typically expressed in TEC units. One TEC Unit (10^{16}electron/m^2) corresponds to about 0.163 meter ionospheric delay at the L1 frequency,
M(h,E)	is the thin-shell mapping function for ionospheric shell height *h* and satellite elevation angle *E* (for the definition of the thin-shell geometric mapping function see e.g. Mannucci et al., [1998] or Komjathy [1997]),
B_i*(lat,lon)*	are horizontal basis functions (based on, for example, bicubic splines or bilinear interpolants) evaluated at the ionospheric pierce point (IPP) – the intersection of the ray path of a signal propagating from the satellite to the receiver with a thin spherical shell – located at latitude *lat* and longitude *lon* on the thin shell,
C_i	are basis function coefficients (real numbers),
b_r, b_s	are the satellite and receiver differential biases, assumed constant over periods of 24 hours or more.

The dependence of vertical TEC on latitude and longitude is parameterized as a linear combination of the two-dimensional basis functions B_i which are functions of solar-geomagnetic longitude and latitude [Mannucci et al., 1998] (We note that the summation in Equation 1 is over all basis functions B_i). Using the carrier phase-leveled ionospheric GPS observables, a Kalman filter simultaneously solves for the instrumental biases and the coefficients C_i which are allowed to vary in time as a random walk stochastic process [Iijima et al., 1999]. The basis functions currently used are based on a bicubic spline technique developed at JPL [Lawson, 1984].

Although the main focus of this research is the investigation of the Brazilian sector, we decided to use a global network of some 230 stations to solve for high precision satellite and receiver differential biases that are used to correct the measurements. Research has shown that the most reliable satellite bias estimates can be achieved when using the data strength of a global network of GPS receivers instead of regional GPS networks [Komjathy, 1997]. We note that the WAAS system itself uses a similar estimation scheme for biases applied over the regional WAAS network.

4. WAAS planar and quadratic fit ionospheric models

In the currently implemented WAAS ionospheric real-time correction algorithm, the vertical ionospheric delay is estimated at each ionospheric grid point (IGP) by constructing a planar fit to a set of (bias-corrected) slant measurements projected to vertical:

$$\mathrm{TEC} = \mathrm{M}(\mathrm{h},\mathrm{E})[\mathrm{a}_0 + \mathrm{a}_1\mathrm{d}_\mathrm{E} + \mathrm{a}_2\mathrm{d}_\mathrm{N}] , \tag{2}$$

where

a_0, a_1, a_2	are the planar fit parameters,
d_E, d_N	are the distances from the IGP to the IPP in the eastern and northern directions, respectively.

Each least squares fit includes all IPPs that lie within a minimum fit radius surrounding the IGP. If the number of IPPs within this minimum radius is less than N_{min}, the fit radius R_{fit} is extended until it encompasses N_{min} points. In this study we do not tabulate data when the fit radius is increased to its maximum value of R_{max} without having reached N_{min} points. R_{max} was chosen to be 2100 km which is the value in the current WAAS implementation. We chose this value to take into consideration of the spacing of reference stations in Brazil and the number of parameters we need to estimate for model comparisons.

In our modified WAAS ionospheric real-time correction algorithm, we estimated the vertical ionospheric delay at each IGP by using a quadratic fit to the slant ionospheric measurements projected to the vertical:

$$\mathrm{TEC} = \mathrm{M}(\mathrm{h},\mathrm{E})\left[\mathrm{a}_0 + \mathrm{a}_1\mathrm{d}_\mathrm{E} + \mathrm{a}_2\mathrm{d}_\mathrm{N} + \mathrm{a}_3\mathrm{d}_\mathrm{E}^2 + \mathrm{a}_4\mathrm{d}_\mathrm{E}\mathrm{d}_\mathrm{N} + \mathrm{a}_5\mathrm{d}_\mathrm{N}^2\right], \tag{3}$$

where

a_3, a_4, a_5	are the additional parameters describing quadratic and cross terms.

In our new WAAS estimation scheme (see Equation 3), we did not solve for the satellite and receiver differential biases. Instead, we used the GIM approach, outlined in Equation 1 to solve for high precision differential biases and calibrated the ionospheric range measurements before applying Equation 3. This is similar to the approach used in WAAS.

5. Data analysis strategy

In our data analysis, we treated every IPP data point as if it were collocated with a WAAS IGP (so-called pseudo IGP approach, see in Komjathy et al. [2002b]). Subsequently, we applied the WAAS planar and quadratic fit ionospheric model algorithms to estimate the vertical ionospheric delays at each of these IPPs, treated as pseudo IGP values. Starting with the set of measurements that contributed to the planar or quadratic fits, we then computed the residual difference between the slant measurements and the estimated slant delays based on the corresponding fit, projecting the vertical TEC from the planar estimate into the line-of-site using the WAAS thin-shell obliquity factor. This residuals analysis provides a measure of the performance of the planar and quadratic fit algorithms in reproducing slant TEC for the user.

6. Data sets

For our data set, we chose quiet and storm days in Brazil, between July 2 and 18, 2000, respectively, using GPS receivers from the International GPS Service [IGS, 2003], and the Brazilian Network for Continuous Monitoring of GPS (RBMC). In Figure 1, we show the Kp indices for a period in 2000 indicating a major geomagnetic storm from July 13 to 16 (St. Swithin s Day).

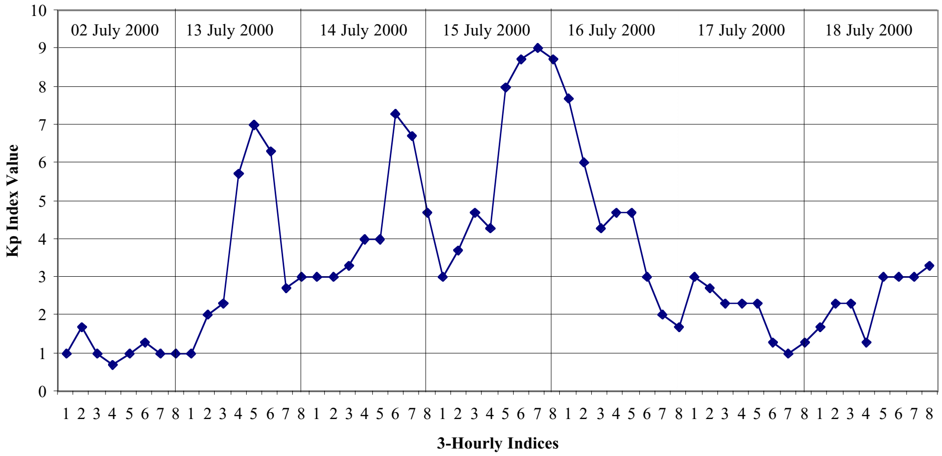

Figure 1. Behavior of Kp indices during the focus period in July 2000.

In Figure 2, we show the distribution of the GPS reference stations in South America for July 16, 2000. The small filled circles in red represent the sites in the vicinity of the equatorial anomaly in Brazil. The ellipse encircles the stations used in our data analysis.

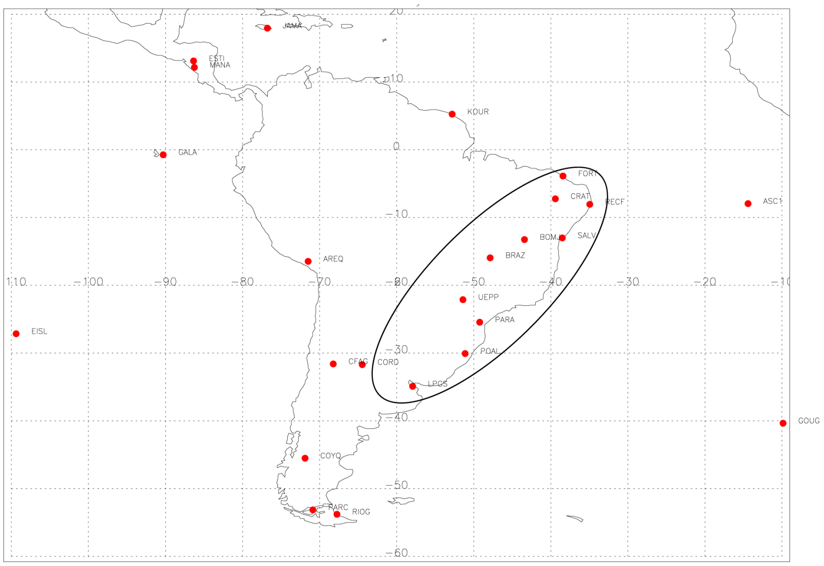

Figure 2. Network of IGS and RBMC stations processed for July 2 to 18, 2000 time interval.

7. Analysis of results

First we calibrated the satellite and receiver differential biases using the GIM method and data from the South American sector. We selected a quiet day (July 2, 2000) and a geomagnetic storm day (July 16, 2000) to illustrate the residuals using the traditional planar fit and the new proposed quadratic fit approaches. In Figure 3, we find that the slant range ionospheric delays for Brazil range between 0 and 52 meters (an elevation cutoff angle of 10 degrees was used throughout this analysis) for the storm day and it ranges between 0 and 25 meters for the quiet day. For plotting, we used 10 stations indicated in Figure 1 with an ellipse. The findings are consistent with our earlier results in Komjathy et al. [2002b] showing quiet and storm day conditions for the March 30 and 31, 2001 period.

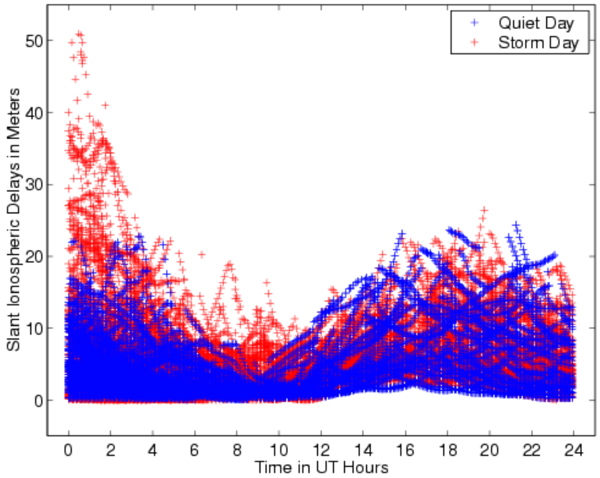

Figure 3. Slant ionospheric delay for the quiet (July 2, 2000) and storm days (July 16, 2000).

In Figures 4 and 5, we plotted the planar and quadratic fit residuals in the slant domain for the quiet and storm days separately. Note that to compute residuals, the fitted vertical TEC value at an IPP location was converted to slant and differenced with the slant TEC measurement. In Figure 4, we show the performance of the planar and quadratic fits for the quiet day. We demonstrate that during post-sunset hours (0 to 6 UT) the temporal variability of the ionosphere is very high. For the quiet day post-sunset hours, the RMS of residuals was reduced from 1.6 meters to 1.4 meters using the quadratic fit. For storm day post-sunset hours, the planar fit residuals are as large as 5 to 15 meters during which time the storm impacted the ionosphere (see Kp index in Figure 1). For this period, the quadratic fit RMS residuals resulted in 2.1 meters compared to 3.2 meters for the planar fit. During daytime (UT 12 to 24), there is also a large variability in the residuals. For the quiet day day-time hours, we achieved an 8 percent RMS improvement using the quadratic fit. For the storm day day-time hours, the improvement is significant: 25 percent.

When comparing the quadratic and planar fit approaches for the quiet day, we achieved an overall 9 percent improvement using the quadratic fit. For the quiet day, we obtained WAAS planar fit residuals less than 13 meters (1.42 meter RMS) in Brazil. When using a quadratic approach we achieved less than 9 meter ionospheric residuals (1.29 meter RMS). For the storm day, an overall 21 percent improvement was obtained. During the storm day, the planar fit residuals were as large as 25 meters with an RMS of 2.16 meters whereas the quadratic fit resulted in less than 11 meters and with an RMS of 1.69 meters. In Figures 4 and 5, we also displayed the 68, 95 and 99.9 percentiles both for planar and quadratic approaches. It is shown that the quadratic fit approach reduces the residuals significantly at the tails of the distribution both for quiet and storm day conditions.

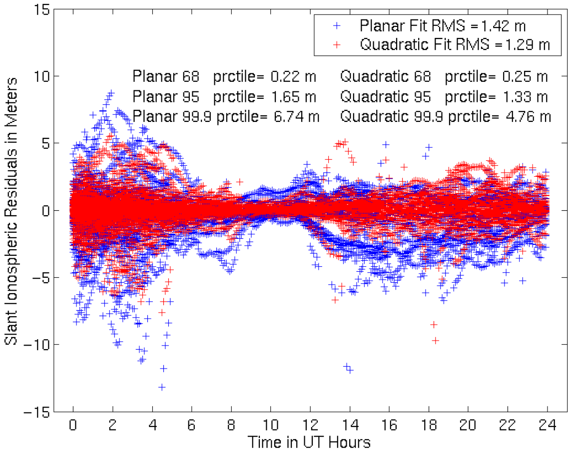

Figure 4. Planar and quadratic fit residuals in Brazil for the quiet day, July 2, 2000.

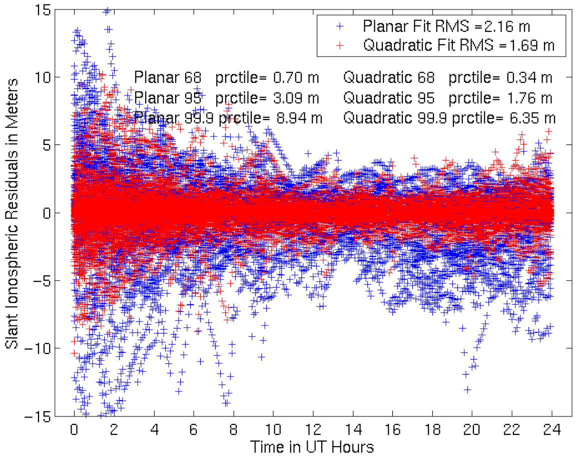

Figure 5. Planar and quadratic fit residuals in Brazil for the storm day July 16, 2000.

In Figure 6, we plot the planar and quadratic fit slant residuals for the quiet day as a function of satellite elevation angle. We can clearly see that we achieved an improvement at low elevation angles over the planar fit approach when using the quadratic fit approach. Figure 7 demonstrates the case for the storm day using planar and quadratic fits. The quadratic fit appears to reduce the residuals both for low and high elevation angles.

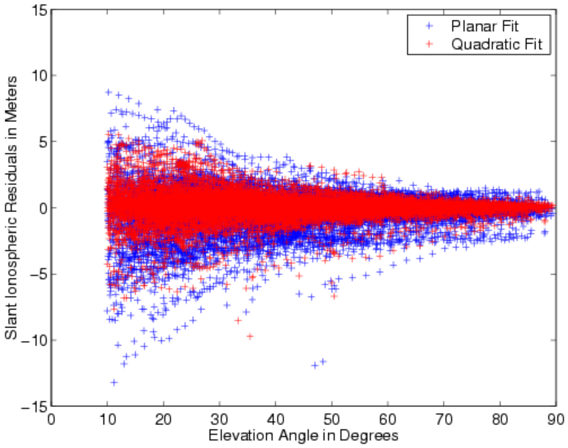

Figure 6. Slant residuals for the <u>*quiet*</u> *day as a function of elevation angle.*

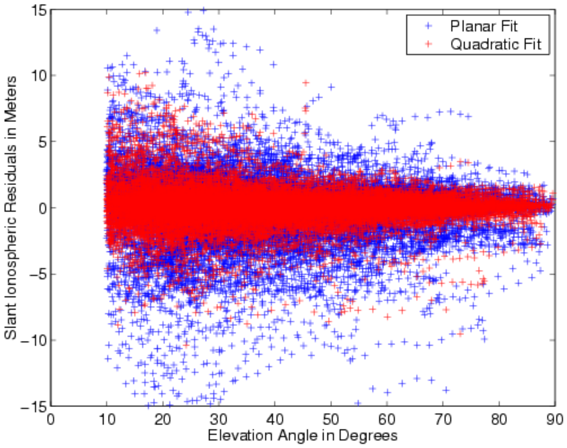

Figure 7. Slant residuals for the <u>*storm*</u> *day as a function of elevation angle.*

In Figures 8 and 9, we display the standard deviation of least squares planar and quadratic fit slant residuals using all data points from the particular fit. We can also consider this as a measure of temporal and spatial decorrelation of the ionosphere. For the quiet day, the quadratic fit shows an improvement in the standard deviation of slant residuals over the planar fit for all local times of the day. For the storm day, we found dramatic improvement especially during the storm time (UT 0 to 6 hours).

In the figures we also indicated the mean of the planar and quadratic fit standard deviations as measures for overall ionospheric decorrelation for quiet and storm days. Note that one has to distinguish between the RMS of slant ionospheric residuals shown in Figures 4-5 and mean fit standard deviations displayed in Figures 8-9. The former may be considered to be a measure of ionospheric model accuracy while the latter can be related to the precision of the ionospheric model. The RMS of slant ionospheric residuals seems to indicate slightly higher numerical values than the mean planar fit sigma values. This must be due to the fact that when computing the slant ionospheric residuals we use the pseudo-IGP approach (i.e., computing the planar and quadratic fit estimates for an IGP after excluding it from the fit) while the fit sigma values include all the measurement residuals. In Figures 8-9, we also displayed percentile values indicating significant improvements in the tails of the distribution.

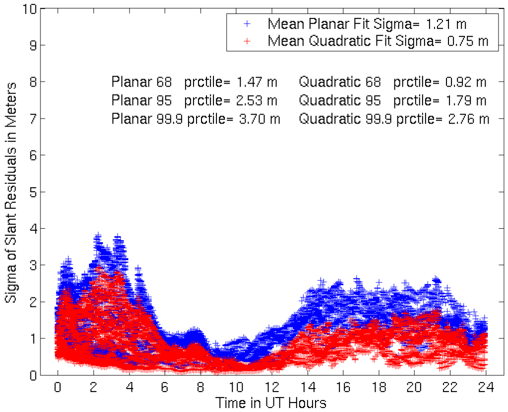

Figure 8. Sigma of slant residuals for the ***quiet*** *day July 2 ,2000.*

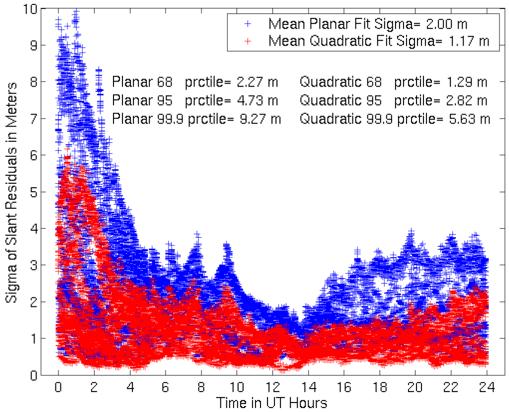

Figure 9. Sigma of slant residuals for the ***storm*** *day July 16, 2000.*

In Table 1, we summarized the statistics for all the days that we processed using the planar and quadratic approaches. From the statistics, we can conclude that we achieved an improvement in the accuracy for all quiet and storm days under investigation. The improvement ranges between 9.1 and 26.6 percent. The smallest improvement 9.1 percent was achieved for the geomagnetic quiet day, July 2, 2000. The largest improvement was obtained for one of the storm days, July 13, 2000. An overall assessment of the statistics indicate that using the quadratic approach over the planar fit, we achieved an average improvement of 20.4 percent. Considering the tails of the distribution (see percentiles) shown in Figures 4 and 5, the improvement of quadratic fit over the planar approach is even more dramatic.

	Planar Fit RMS in Meters	**Quadratic Fit RMS in Meters**	**Improvement in Percent**
7/2/00	1.42	1.29	9.1
7/13/00	1.65	1.21	26.6
7/14/00	1.60	1.19	25.6
7/15/00	2.03	1.57	22.6
7/16/00	2.16	1.69	21.7
7/17/00	1.40	1.08	22.8
7/18/00	1.66	1.39	16.2
Average	1.70	1.35	20.5

Table 1.RMS of daily residuals in meters.

Implications for LNAV/VNAV availability. In this research, we have investigated ionospheric range errors in the low-latitude Brazilian sector for quiet and storm days. Based on this limited data set, we were able to achieve 1.4 meter RMS ionospheric slant delay planar fit residuals for a quiet day and 2.2 meters for a storm day. In our earlier investigation documented in Komjathy et al. [2002b] we found that for CONUS we were able to achieve a better than 0.5 meter RMS ionospheric slant delay planar fit residuals for the quiet day and 0.8 meter for the storm day in CONUS. Based on our current results, it appears that the planar residuals in Brazil are a factor of 2.8 larger than in the

CONUS for quiet and a factor of 2.7 for storm days. Using the quadratic fit, we found that the residuals in Brazil are still larger by a factor of 2.5 than in the CONUS planar fits for quiet day and by a factor of 2.1 for the storm day.

This has major implications for availability of the initial WAAS Lateral Navigation/Vertical Navigation service (LNAV/VNAV). The user determines, in real-time, the level of navigation service available based on the broadcast grid ionosphere vertical errors (GIVEs) and other information. GIVE values represent 3.29-sigma bounds on vertical ionosphere range error at each ionospheric grid point. Service volume model studies for WAAS have shown that high availability of LNAV/VNAV service is possible when a significant majority of the broadcast GIVEs are in the range 3-6 meters. This performance is expected for WAAS.

At low latitudes, the GIVEs must be increased to cover the larger ionospheric range errors expected. Increased planar fit residuals by a factor of three are likely to result in a substantial number of GIVEs above 6 meters. Due to GIVE quantization in the broadcast message, computed GIVEs above 6 meters are transmitted as 15-meter bounds to the user. It is clear that LNAV/VNAV service will be unavailable if several of the user s satellite links are associated with GIVEs of 15 meters or more.

We expect that the WAAS planar fit algorithm applied to Brazil will result in significantly reduced availability of LNAV/VNAV service, particularly near solar maximum during daytime and evenings.

8. Conclusions

In this paper, we have compared the performance of the WAAS ionospheric planar-fit and the new quadratic-fit approaches in the low latitude region where the temporal and spatial variation of the ionosphere is the highest in the world. We used data from the network of dual-frequency GPS receivers in and around Brazil. Unbiased line-of-site TEC ground-truth data were generated using JPL s Global Ionospheric Mapping (GIM) software. Using the truth data, the WAAS planar fit and the quadratic algorithms were evaluated by treating each observation as representing a WAAS ionospheric grid point (IGP) and computing the planar and quadratic fit estimates for that IGP after excluding it from the fit.

We found slant ionospheric range delays up to 55 meters for Brazil. For the quiet day, we obtained WAAS planar fit residuals less than 13 meters (1.42 meter RMS) in Brazil. When using a quadratic approach we obtained less than 9 meter (1.29 meter RMS) slant residuals. For the storm day, the planar fit residuals were as large as 25 meters (2.16 meter RMS) whereas the quadratic fit resulted in less than 11 meters (1.69 meter RMS). When considering all quiet and storm days, we found that the quadratic fit approach reduced the residuals by an average of 20 percent.

It appears that the Brazil planar fit residuals are a factor of three larger than those for CONUS. The quadratic fit residuals are also increased by at least a factor of 2. The main contributing factors of the large residuals in Brazil appear to be the high spatial variability of the spatial gradients, the large absolute slant TEC and the errors in the ionospheric mapping function. This will likely result in a substantial number of GIVEs about 6 meters, significantly reducing LNAV/VNAV availability service in Brazil.

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