On the Ionospheric Impact of Recent Storm Events on Satellite-Based Augmentation Systems in Middle and Low-Latitude Sectors

Attila Komjathy, Lawrence Sparks, Anthony J. Mannucci and Xiaoqing Pi

Jet Propulsion Laboratory/ California Institute of Technology
M/S 238-600, 4800 Oak Grove Drive, Pasadena, CA 91109

BIOGRAPHIES

Attila Komjathy is currently a staff member of the Ionospheric and Atmospheric Remote Sensing (IARS) Group of the Tracking Systems and Applications Section at Jet Propulsion Laboratory (JPL), specializing in remote sensing techniques using the Global Positioning System. Prior to his joining JPL in July 2001, he worked on the utilization of GPS reflected signals as a Research Associate at the University of Colorado's Center for Astrodynamics Research. He received his Ph.D. from the Department of Geodesy and Geomatics Engineering of the University of New Brunswick, Canada in 1997.

Lawrence Sparks is a senior member of the IARS Group at JPL. He received his Ph.D. in Applied Physics from Cornell University. His published research has spanned fields including fusion plasma physics, solar magneto-hydrodynamics, atmospheric radiative transfer, and ionospheric modeling. He is currently working on applications of GPS to ionospheric science.

Anthony J. Mannucci is supervisor of the IARS Group at JPL. He has developed ionospheric calibration systems for deep space tracking and Earth science applications. He works with the Federal Aviation Administration on the Wide Area Augmentation System differential GPS implementation and is a member of the international ionospheric working group for Satellite-Based Augmentation Systems (SBAS). He obtained a Ph.D. in Physics from U.C. Berkeley in 1989.

Xiaoqing Pi is a senior member of the Ionospheric and Atmospheric Remote Sensing Group at JPL. He is also an associate research professor in the Mathematics Department of the University of Southern California. He received a Ph.D in astronomy from Boston University. He has been involved in the research and development of ionospheric applications of GPS, ionospheric modeling, and ionospheric storms, irregularities and scintillation. He is currently working on ionospheric data assimilation models, satellite-based augmentation systems, and ionospheric scintillation of GPS signals.

ABSTRACT

Ionospheric correction algorithms have been characterized extensively for the mid-latitude region of the ionosphere where benign conditions usually exist. The United States Federal Aviation Administration s (FAA) Wide Area Augmentation System (WAAS) for civil aircraft navigation is focused primarily on the Conterminous United States (CONUS). Other Satellite-Based Augmentation Systems (SBAS) include the European Geostationary Navigation Overlay Service (EGNOS) and the Japanese Global Navigation Satellite System (MSAS). Other parts of the world, such as the South American region or India, provide a more serious challenge to addressing the impact of the ionosphere on navigation using SBAS. At equatorial latitudes, geophysical conditions produce the so-called Appleton-Hartree (equatorial) anomaly, which results in significantly larger ionospheric range delays and larger spatial gradients in these delays than are observed in the CONUS or European sectors.

In this paper, we use GPS measurements obtained on geomagnetic storm days to perform a quantitative assessment of WAAS-type ionospheric correction algorithms in other parts of the world such as low-latitude Brazil and mid-latitude Europe. For the study, we access a world-wide network of 400+ dual-frequency GPS receivers. The network includes: 1) the Continuously Operating Reference Sites (CORS) in the United States;

2) stations in and near South America as part of the Brazilian Network of Continuous Monitoring of GPS (RBMC), operated by the Brazilian Institute of Geography and Statistics (IGBE); and (3) European and CONUS sites included in the International GPS Service (IGS) global network. Data sets have been selected to include 18 quiet and storm days upon which the WAAS ionospheric threat model is based. To provide ground-truth and calibrate GPS receiver and transmitter inter-frequency biases, we process the GPS data using Global Ionospheric Mapping (GIM) software developed at the Jet Propulsion Laboratory to compute calibrated high resolution observations of ionospheric total electron content (TEC).

In this research, we investigate major storm events of the last few years and evaluate their impact on WAAS ionospheric model performance in Brazil, Europe and CONUS. These storms include the worst-case CONUS storms such as those on July 15, 2000, and March 31, 2001; we investigate their impact on SBAS in Europe and Brazil. Results indicate that lesser known storms such as the one on April 5, 2000 had a more significant impact in Europe than a near worst-case storm in CONUS. Furthermore, in terms of planar fit residuals, we provide additional evidence that there is little difference between quiet and storm time behavior over Brazil. We find that the Brazilian planar fit residuals are 2 to 4 times higher (RMS) than those for Europe and CONUS.

INTRODUCTION

The Wide-Area Augmentation System (WAAS) developed for the Conterminous United States (CONUS) is only one of the several Space-Based Augmentation Systems (SBAS) under consideration worldwide. Other SBAS developments are under way in Europe, Japan, India and Brazil.

Relatively benign ionospheric conditions in the mid-latitude CONUS region are compatible with accurate ionospheric range corrections for WAAS. Providing ionospheric corrections for Brazil is significantly more challenging, since ionospheric range delays and range delay spatial gradients are among the largest in the world, even in the absence of ionospheric storms (during infrequent ionospheric storms, even mid-latitude regions present challenging conditions). In short, the ionosphere in the Brazilian sector shows significantly different behavior from that of the mid-latitude sector.

The ionosphere has been extensively studied to support WAAS at the CONUS sector. The published literature discussing ionospheric corrections for WAAS in the CONUS is extensive; see e.g. Enge et al., [1996], WAAS MOPS [1999], Walter et al., [2000] and Sparks et al., [2002]. Various alternative ionospheric correction algorithms have been presented by e.g., Hansen et al., [1997], Sparks et al., [2000] and Blanch et al., [2002]. A potential application of WAAS algorithms to Brazil has been investigated by Klobuchar et al., [2001] using simulated data. The authors have shown large differences between modeled slant TEC from LowLat model by Anderson [1973] and values interpolated from ionospheric grid points. Klobuchar et al., [2002] have revealed differences between modeled slant TEC values and those obtained from interpolating from ionospheric grid points (IGPs) can be up to 14 meters. The temporal and spatial variability of the low-latitude ionosphere was studied in the context of ionospheric storms by Dehel and Corbelli [2002] and Fedrizzi et al., [2001] using a network of dual-frequency GPS receivers in Brazil. Lejeune et al., [2002] investigated spatial decorrelation errors in the equatorial region which were found to be an order of magnitude larger than those found in the CONUS, computed using LowLat simulations of the equatorial ionosphere. Furthermore, several meter level errors were obtained from the standard slant-to-vertical conversion technique. In a paper by Doherty et al., [2002], scintillation, depletion and large gradient TEC issues were discussed in the equatorial region. Investigating the possible application of the current WAAS algorithm using actual GPS-derived TEC measurements is the natural progression of the previous studies and therefore the main focus of our subsequent papers.

In Komjathy et al., [2002b], we assessed the WAAS planar fit algorithm in the equatorial region where the spatial gradients and the absolute slant TEC are known to be the highest in the world. We found that in Brazil the dominant error source for the WAAS planar fit algorithm is the inherent spatial variability of the equatorial ionosphere, with ionospheric slant range delay residuals as high as 15 meters and root-mean square (RMS) residuals for the quiet day of 1.9 meters. This compares to a quiet-day maximum residual of 2 meters in CONUS, and 0.5 meter RMS. We revealed that ionospheric gradients in Brazil are, on average, at the level of 2 meters over 100 km. Contrary to results obtained for CONUS, we discovered that a major ionospheric storm (March 31, 2001) had small impact on the planar fit residuals in Brazil.

In a subsequent study by Komjathy et al., [2003], we have compared the performance of the WAAS ionospheric planar-fit and a new quadratic-fit approach in the low latitude region where the temporal and spatial variation of the ionosphere is the highest of all latitude regions. We found slant ionospheric range delays up to 55 meters for Brazil. For a quiet day, we obtained WAAS planar fit residuals less than 13 meters (1.42 meter RMS) in Brazil. When using a quadratic approach we obtained slant residuals less than 9 meters (1.29 meter RMS). For a storm day, the planar fit residuals were as large as 25 meters (2.16 meter RMS) whereas the quadratic fit residuals were always less than 11 meters (1.69 meter RMS). When considering all quiet and storm days, we found that the quadratic fit approach reduced the residuals by an average of 20 percent. We found that the Brazil planar fit residuals are a factor of three larger than those for CONUS. The quadratic fit residuals are also increased by at least a factor of 2. The main contributing factors of the large residuals in Brazil appear to be the high spatial variability of the spatial gradients, the large absolute slant TEC and the errors in the ionospheric mapping function (i.e., slant to vertical conversion).

In this research, we investigate a larger and more comprehensive data set including the same major storm events that the WAAS CONUS threat model is based on. To establish a background for this work, we first review the estimation method that uses a global network of 230 GPS sites to solve for inter-frequency biases (nuisance parameters) in the GPS satellites and receivers, in order to provide ground truth data for the analysis. Subsequently, we briefly review the standard WAAS planar fit algorithm used to estimate the vertical ionospheric range delay at fixed latitude/longitude locations known as ionospheric grid points (IGPs). We examine the implications of using the currently adopted WAAS algorithm in Brazil, Europe and CONUS to compare ionospheric range residuals.

GIM BIAS ESTIMATION STRATEGY

To provide ground-truth, we use the Global Ionospheric Mapping (GIM) software developed at the Jet Propulsion Laboratory [Mannucci et al., 1998] to compute high precision slant ionospheric delay. This algorithm removes the satellite and receiver differential biases from the ionospheric observables that are generated from carrier-phase data adjusted to match the ionospheric delay based on dual-frequency pseudoranges. The estimation of the satellite and receiver biases has been published previously but we will review it here briefly.

Ionospheric measurements from a GPS receiver can be modeled with the well-known single-shell ionospheric model using the following observation equation [see e.g. Mannucci et al., 1999 and Komjathy et al., 2002a]:

$$TEC = M(h,E)\sum_i C_i B_i(lat,lon) + b_r + b_s \ , \qquad (1)$$

where

TEC	is the slant Total Electron Content measured by the linear combination of the GPS dual-frequency carrier phase and pseudorange ionospheric observables, typically expressed in TEC units; one TEC Unit (10^{16}electron/m^2) corresponds to about 0.163 meter ionospheric delay at the L1 frequency;
$M(h,E)$	is the thin-shell mapping function for ionospheric shell height h and satellite elevation angle E (for the definition of the thin-shell geometric mapping function see e.g. Mannucci et al., [1998] or Komjathy [1997];
$B_i(lat, lon)$	are horizontal basis functions (based on, for example, bicubic splines or bilinear interpolants) evaluated at the ionospheric pierce point (IPP) − the intersection of the ray path of a signal, propagating from the satellite to the receiver, with a thin spherical shell − located at latitude *lat* and longitude *lon* on the thin shell;
C_i	are basis function coefficients (real numbers);
b_r, b_s	are satellite and receiver differential biases, assumed constant over periods of 24 hours or more.

The dependence of vertical TEC on latitude and longitude is parameterized as a linear combination of the two-dimensional basis functions B_i which are functions of solar-geomagnetic longitude and latitude [Mannucci et al., 1998] (We note that the summation in Equation 1 is over all basis functions B_i). Using the carrier phase-leveled ionospheric GPS observables, a Kalman filter simultaneously solves for the instrumental biases and the coefficients C_i, which are allowed to vary in time as a random walk stochastic process [Iijima et al., 1999]. The basis functions currently used are based on a bicubic spline technique developed at JPL [Lawson, 1984].

Although the main focus of this research is the comparison between CONUS, Europe and Brazilian sectors, we have employed a global network of some 230 stations to solve for high precision satellite and receiver differential biases that are used to correct the measurements. Research has shown that the most reliable satellite bias estimates can be achieved when using the data strength of a global network of GPS receivers instead of regional GPS networks [Komjathy, 1997]. We note that the WAAS system itself uses a similar estimation scheme for biases applied over the regional WAAS network.

WAAS PLANAR FIT IONOSPHERIC MODEL

In the currently implemented WAAS ionospheric real-time correction algorithm, the vertical ionospheric delay is estimated at each ionospheric grid point by constructing a planar fit of a set of (bias-corrected) slant measurements projected to vertical:

$$TEC = M(h,E)\left[a_0 + a_1 d_E + a_2 d_N\right] \quad , \qquad (2)$$

where

a_0, a_1, a_2 are the planar fit parameters;

d_E, d_N are the distances from the IGP to the IPP in the eastern and northern directions, respectively.

Each least squares fit includes all IPPs that lie within a minimum fit radius surrounding the IGP. If the number of IPPs within this minimum radius is less than N_{min}, the fit radius R_{fit} is extended until it encompasses N_{min} points. In this study we do not tabulate data when the fit radius reaches its maximum value of R_{max} without having encompassed N_{min} points. R_{max} is chosen to be 2100 km which is the value in the current WAAS implementation. We choose this value to take into consideration of the spacing of reference stations in Brazil and the number of parameters we need to estimate for model comparisons.

In our WAAS estimation scheme (see Equation 2), we do not solve for the satellite and receiver differential biases. Instead, we use the GIM approach, outlined in Equation 1, to solve for high precision differential biases, and we calibrate the ionospheric range measurements before applying Equation 2. This is similar to the approach used in WAAS.

DATA ANALYSIS STRATEGY

In our data analysis, we treat every IPP data point as if it were collocated with a WAAS IGP (a so-called pseudo-IGP approach). Subsequently, we apply the WAAS planar fit ionospheric model algorithm to estimate the vertical ionospheric delays at each of these IPPs, treated as a pseudo-IGP. Starting with the set of measurements that contribute to the planar fit, we then compute the residual difference between the slant measurements and the estimated slant delays based on the planar fit, projecting the vertical TEC from the planar estimate into the line-of-site using the WAAS thin-shell obliquity factor. This residuals analysis provides a measure of the performance of the planar fit algorithm in reproducing slant TEC for the user.

DATA SETS

For our test data set, we choose both quiet and storm day, using GPS receivers from the Continuously Operating Reference Stations (CORS) network, maintained by the US National Geodetic Survey [CORS, 2002], the International GPS Service [IGS, 2003], and the Brazilian Network for Continuous Monitoring of GPS (RBMC).

In Figure 1, we show the global distribution of the GPS reference stations for a typical day of the data set. The small filled circles represent the 230 sites that are used to provide unbiased line-of-sight TEC ground-truth data. The larger circles in blue indicate the CONUS, European and Brazilian sectors for which stations are used for the data analysis.

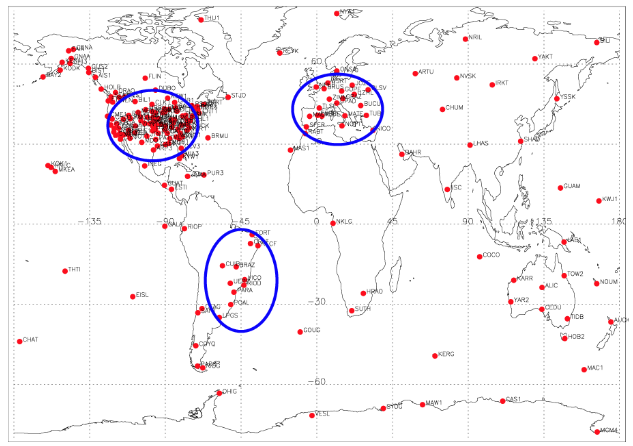

Figure 1. Network of CORS, IGS and RBMC stations processed for the 18 day focus period.

The 18 days we have selected coincide with the focus period for which the WAAS ionospheric threat model for CONUS was derived. In Figure 2, we show the 3-hour Kp indices for a period between January 2000 and September 2002 encompassing major storm evens.

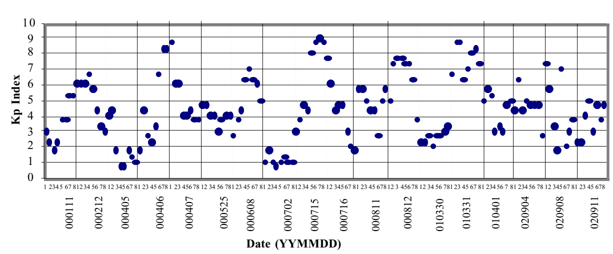

Figure 2. Behavior of Kp index during the selected 18 days between January 2000 and September 2002.

ANALYSIS OF RESULTS

First we calibrate the satellites and receiver differential biases using the GIM method and data from the CONUS, European and South American sectors. We select a quiet (April 5, 2000) and two subsequent storm days (April 6-7, 2000) to illustrate residuals using the traditional planar fit approach.

Comparison of European and WAAS CONUS residuals for a quiet day. For the comparison we use planar fit residuals between 30 and 60 degrees geomagnetic latitudes making sure that we use about the same number of observations covering the same geomagnetic latitude band. In Figure 3, we plot the European and CONUS planar fit residuals in the slant domain for the quiet day as a function of local time. Note that, to compute residuals, the fitted vertical TEC value at an IPP location was converted to slant and differenced with the slant TEC measurement. It is clear that the European and CONUS residuals are very similar for the quiet day of April 5, 2000. For the quiet day, we obtain WAAS planar fit residuals less than 7.2 meters (0.52 meter RMS). For Europe, we achieve less than 9.8 meter ionospheric residuals (0.66 meter RMS). In the figure, we have also displayed the 68, 95 and 99.9 percentiles both for Europe and CONUS approaches. It is shown that the WAAS CONUS has smaller residuals at the tails of the distribution.

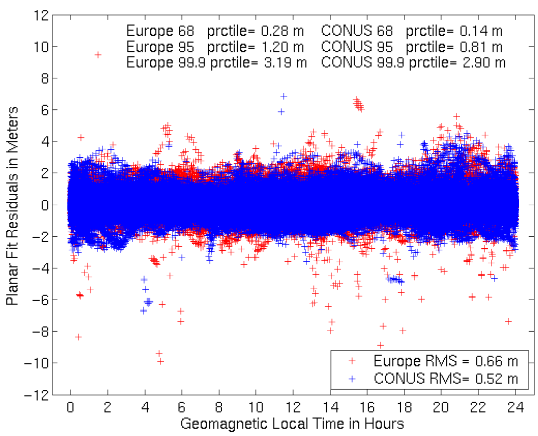

Figure 3. Planar fit residuals for WAAS CONUS and Europe for the *quiet* day of April 5, 2000.

Comparison of European and WAAS CONUS residuals for storm days. In Figure 4, we show the planar fit residuals for the subsequent storm day both for the European and CONUS regions. The WAAS CONUS residuals demonstrate that the storm affected the ionosphere during day-time hours. On the other hand, it is indicated that the storm affected the European sector during post-sunset hours. For the entire day, the European fit RMS residuals resulted in 0.9 meter compared to the 0.68 meter for the WAAS CONUS fit. It appears that for April 6th, the CONUS fit has smaller residuals at the tails of the distribution.

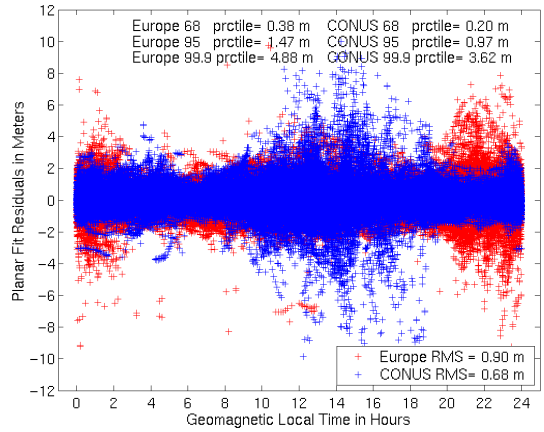

Figure 4. Planar fit residuals for WAAS CONUS and Europe for the *storm* day of April 6, 2000.

Figure 5 indicates an opposite trend. The subsequent storm day appears to have affected the WAAS CONUS sector more than the European sector. In the CONUS sector we see that the temporal variability of the ionosphere is high throughout the day. For the storm day, we obtain WAAS CONUS planar fit residuals of 0.93 meter RMS compared to the corresponding European value of 0.61 meter (RMS). The percentiles indicate that the tails of the residual distribution are now wider for the CONUS sector.

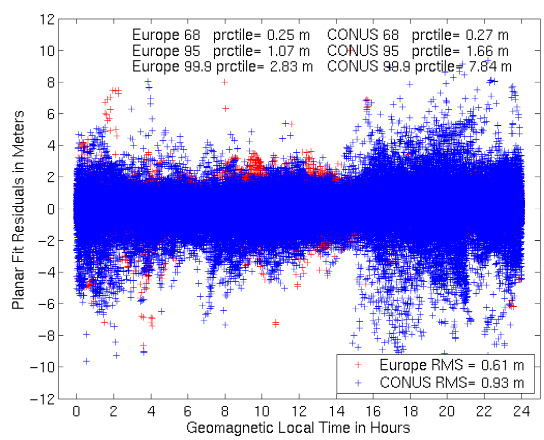

Figure 5. Planar fit residuals for WAAS CONUS and Europe for the *subsequent storm* day of April 7, 2000.

Comparison of Brazilian residuals between quiet and storm days. In Figure 6, we plot the Brazilian planar fit residuals for the quiet (April 5) and one storm day (April 6). We find planar fit residuals during day-time hours at the 8 meter level. During post-sunset hours we obtain residuals as large as 20 meters. It is striking that the quiet and storm time equatorial ionospheric planar fit residuals behave similarly. We do not see a major impact of the storm compared to quiet time behavior. This finding corresponds to our earlier results where we investigated the March 31, 2000 storm event. The RMS of planar fit residuals is somewhat higher for the storm days (2.4 meters) compared to the quiet day (2.07 meters). The distribution of the residuals is wider for the storm day for all three percentiles we investigated.

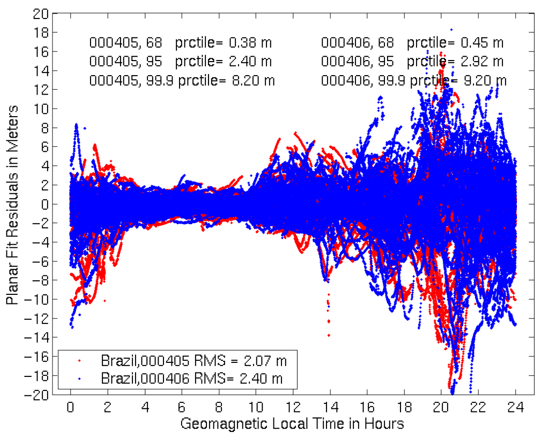

Figure 6. Brazilian planar fit residuals for quiet and storm days.

Figure 7 displays the Brazilian planar fit residuals as a function of elevation angle. The slight difference in the RMS of residuals between the quiet and storm days in Brazil can be explained by the more complex spatial structures in the ionosphere during storm. As a consequence, large residuals even at high elevation angles are seen during the storm days.

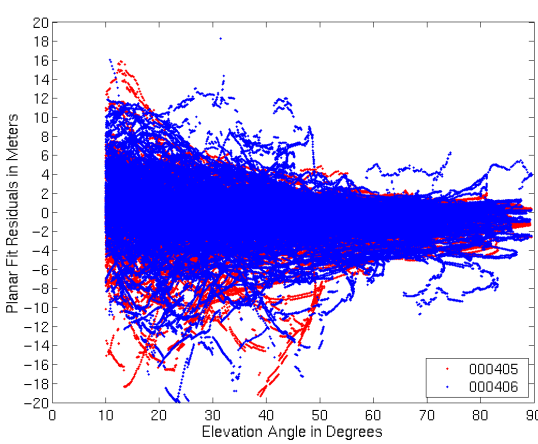

Figure 7. Brazilian planar fit residuals for quiet (April 5) and storm days (April 6) as a function of elevation angle.

In Figure 8, a summary plot is presented showing the overall RMS of planar fit residuals separately for CONUS, Europe and Brazilian sectors and for different days. It is demonstrated that 14 out of 18 quiet and storm days, the European RMS of planar fit residuals are higher than those for the WAAS CONUS residuals. We also find that the Brazilian planar fit residuals are less than 2.5 meters (RMS) for both quiet and storm days. It is also seen that the Brazilian planar fit residuals are on average, a factor of 2 to 4 times larger than the WAAS CONUS or European residuals.

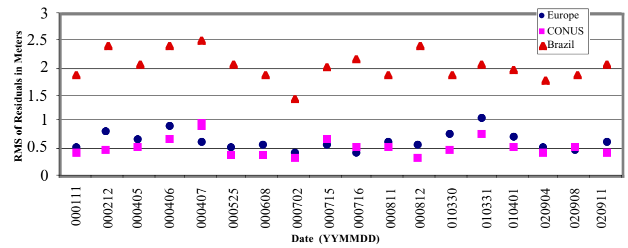

Figure 8. Summary of results for CONUS, Europe and Brazil data processing.

Implications for LNAV/VNAV availability in the low latitude sector. In this research, we have investigated ionospheric range errors in the low-latitude Brazilian sector for quiet and storm days. Based on this data set, we achieve 1.4 meter RMS ionospheric slant delay planer fit residuals for a quiet and 2.5 meters for storm day. Based on our current and earlier results, it appears that the planar fit residuals in Brazil are a factor of 2 to 4 larger than in the CONUS or Europe.

At low latitudes, the GIVEs (Grid Ionospheric Vertical Error) must be increased to cover the larger ionospheric range errors expected. Planar fit residuals increased by a factor of three are likely to result in a substantial number of GIVEs above 6 meters. Due to GIVE quantization in the broadcast message, computed GIVEs above 6 meters are transmitted as 15-meter bounds to the user. It is clear that LNAV/VNAV service will be unavailable if several of the user s satellite links are associated with GIVEs of 15 meters or more.

We expect that the WAAS planar fit algorithm applied to Brazil will result in significantly reduced availability of LNAV/VNAV service, particularly near solar maximum during daytime and evenings.

CONCLUSIONS AND FUTURE RESEARCH

In this paper, we have compared the performance of the WAAS CONUS, European and Brazilian ionospheric planar-fit residuals. In the case of the low latitude region, it is widely accepted that the temporal and spatial variation of the ionosphere is here the highest in the world. We have used data from the network of dual-frequency GPS receivers using IGS, CORS and RBMC stations in CONUS, Europe and Brazil. Unbiased line-of-site TEC ground-truth data have been generated using JPL s Global Ionospheric Mapping (GIM) software. Using the truth data, the planar fit has been evaluated by treating each observation as representing a WAAS ionospheric grid point (IGP) and computing the planar fit estimates for that IGP after excluding it from the fit.

In the comparison of the European and WAAS CONUS planar fit residuals, we find that the European ionosphere shows as much storm-time variation as it displays in the CONUS region. In 14 out of 18 cases, we obtain a larger RMS of planar fit residuals for Europe than for CONUS.

We have investigated 18 days of quiet and storm-time data from Brazil. From this larger data set, we have confirmed for the planar fit residuals that there is little difference between quiet and storm time behavior. We also find that on average the Brazilian planar fit residuals are 2 to 4 times higher (RMS) than those for the CONUS or Europe. The main contributing factors of the large residuals in Brazil appear to be the high spatial variability of the spatial gradients, the large absolute slant TEC, and the errors in the ionospheric mapping function. This will likely result in a substantial number of GIVEs above 6 meters, significantly reducing LNAV/VNAV availability service in Brazil.

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