A New Ionospheric Model for Wide Area Differential GPS: The Multiple Shell Approach

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BIOGRAPHIES

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Brian D. Wilson, a senior member of the IARS Group at JPL, has been studying the ionosphere using GPS for ten years and is a co-developer of the GPS-based Global Ionospheric Mapping (GIM) technique. His recent work includes: improving and validating GIM TEC maps, global ionospheric calibration of single-frequency ocean altimetry missions, real-time global WADGPS, and the development of a new Global Assimilative Ionosphere Model (GAIM) which uses a first-principles physics model and formal data assimilation techniques (4DVAR and Kalman filtering) to solve for the global 3D plasma density using GPS, ionosonde, UV imaging, and other satellite data.

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ABSTRACT

Global Ionospheric Modeling (GIM) algorithms have been recently enhanced to solve for electron content distributions on multiple horizontal grids distributed vertically (multiple shells), instead of using a single grid at a fixed height (single shell). We are assessing this new ionospheric model for application in Wide Area

Differential GPS (WADGPS) systems over the coterminous United States (CONUS). The additional parameters from multiple vertical shells allow GIM to better model the height variation of ionospheric electron density along the GPS raypaths, and accommodate significant diurnal height variations of the ionosphere that are ignored in a fixed-height single shell approach. This new model is conceptually a simple extension of several existing WADGPS algorithms and may offer benefits similar to various forms of ionospheric tomography. We compare solutions that model the ionosphere as a correlated random-walk stochastic process (the standard GIM approach) using multiple shells, with an older strategy assuming the ionospheric centroid height to be fixed. It is shown that the multi-shell approach improves slant ionospheric delay accuracy at low elevation angles by about 0.2 meter RMS on L1 and reduces systematic error in the GPS inter-frequency bias estimates by a factor of 2−4.

INTRODUCTION

Currently the largest error source in single-frequency GPS positioning is the propagation delay caused by ionospheric refraction. The disturbing influences of the temporally and spatially varying ionization of the ionosphere have great impact on satellite navigation using GPS. Dual-frequency observations can be used to eliminate almost all of the ionosphere's effect. To correct data from a single-frequency GPS receiver for the ionospheric effect, there are several techniques that one can use.

We can ignore the effect and live with the consequences. However, the effect can be quite severe since the measurement error caused by the ionosphere can be as much as 50 meters on L1. With a Position Dilution of Precision (PDOP) value typically less than 3, the positioning error caused by the ionosphere can be as much as 150 meters during periods of high solar activities such as the current year of 2002.

Another way of mitigating the ionospheric effect is to use various data processing techniques such as forming double differences of the GPS observables. For point positioning purposes, we usually do not have the luxury of doing this. However, we can use other empirical or physics-based first principle models to mitigate about 50% of the RMS ionospheric delay [Langley, 1996].

It is also possible to use wide area differential GPS (WADGPS) corrections to mitigate ionospheric errors (see e.g., Wells et al., 1987; Parkinson et al., 1996). To provide accurate ionospheric delay corrections for single frequency GPS users, WADGPS systems broadcast ionospheric delay estimates derived from reference networks of dual-frequency GPS receivers. Global receiver networks have been used for many years to measure and map ionospheric total electron content (TEC) and hence ionospheric delays, on global scales. In particular, Global Ionospheric Mapping (GIM) software developed at the NASA Jet Propulsion Laboratory uses observations from about 100 GPS sites to compute global maps of vertical TEC with 15-minute time resolution and about 5-degree spatial resolution. The vertical variation of the ionospheric electron density is represented by a simplified, predetermined form consisting of a constant density slab at fixed height with exponential tails [Ho et al., 1996; Mannucci et al., 1998]. A novel approach by Sparks et al. [2000] describes a WADGPS ionospheric model to simultaneously retrieve multiple parameters from the GPS data representing integrated quantities in addition to vertical ionospheric delay. Regional TEC maps such as those produced by Jakowski et al. [1998], Schaer et al. [1999], and Fedrizzi et al. [2001] could also be used to provide WADGPS-type ionospheric corrections. As a viable alternative approach, Hansen et al. [1997] described a mathematical framework to use tomography for providing WADGPS ionospheric corrections by integrating electron densities in the vertical dimension.

In this paper we report on recent improvements we have made to the single shell ionospheric model often used in providing WADGPS corrections. We have made a simple extension to the model by including two more shells to solve for horizontal basis functions on three separate shells. The new model has been validated by excluding a handful of GPS sites from the solution and then predicting the slant ionospheric delay for the stations removed. As a measure of prediction accuracy, we form the RMS of differences between the predicted and measured slant ionospheric delays.

ESTIMATION STRATEGY

The ionospheric measurements from a GPS receiver can be modeled with the commonly used single-shell ionospheric model using the observation equation:

$$TEC = M(h,E)\sum_i C_i B_i(lat,lon) + b_r + b_s \ , \tag{1}$$

where

TEC is the slant Total Electron Content measured by the linear combination of the GPS dual-frequency carrier phase and pseudorange ionospheric observables, where 1 TEC Unit (10^{16}electron/m^2) corresponds to about 0.163

meter ionospheric delay on the L1 frequency,

$M(h,E)$ is the thin-shell mapping function for ionospheric shell height h and satellite elevation angle E (for the definition of the thin-shell geometric mapping function see e.g. Mannucci et al. [1998] or Komjathy [1997],

B_i are the horizontal basis functions (bicubic splines (C^2) or triangular interpolation (TRIN)) at the ionospheric pierce point – the intersection of the ray path of a signal propagating from the satellite to the receiver with a thin spherical shell defined by geodetic latitude *lat* and longitude *lon* on the thin shell,

C_i are the basis function coefficients,

b_r, b_s are the satellite and receiver differential biases.

The new, modified model includes three distinct shells described by the following observation equation:

$$\begin{aligned}TEC = {} & M(h_1,E)\sum_i C_{1i}B_i(lat_1,lon_1) + \\ & M(h_2,E)\sum_i C_{2i}B_i(lat_2,lon_2) + \\ & M(h_3,E)\sum_i C_{3i}B_i(lat_3,lon_3) + b_r + b_s\,,\end{aligned} \tag{2}$$

where

$M(h_1,E)$ is the thin shell mapping function for shell 1, etc.,

C_{1i} are the basis function coefficients solved for in the filter, indexed by horizontal (i) and vertical (1,2,3 for three shells) indices.

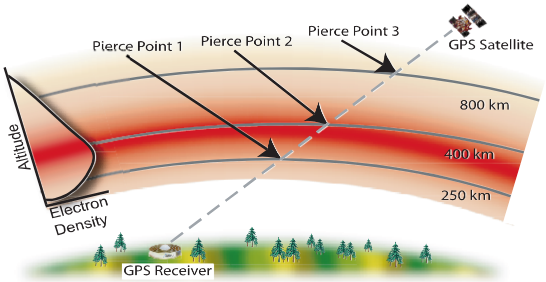

Figure 1. Illustration for the multi-shell model.

Figure 1 illustrates the three shells set at 250, 400 and 800 km. We found that the combination of three shells was best in reducing the ionospheric residuals [Komjathy et al., 2002]. As shown in Figure 1, the line-of-sight vector pierces the ionospheric shells at three separate points. The slant TEC data are converted to the vertical using the obliquity function $M(h_i,E)$ computed separately for each shell. The vertical TEC dependence on latitude and longitude is parameterized as a linear combination of basis functions B_i with coefficients C_i as a function of solar-geomagnetic longitude and latitude. Using the phase-leveled ionospheric observable, the Kalman filter simultaneously solves for the instrumental biases and the coefficients C_i which are allowed to vary in time as a random walk stochastic process [Iijima et al., 1999]. The basis functions currently used are based on a bicubic spline technique developed at JPL [Lawson, 1984].

DATA SETS

For our test data set, we chose a quiet day and a storm day, 5 April and 6 April 2000 respectively, using GPS receivers from the Continuously Operating Reference Stations (CORS) network maintained by the US National Geodetic Survey (CORS, 2002) and the International GPS Service (IGS, 2002).

In Figure 2. we show the locations of the GPS reference stations. Circles represent the 90 CORS sites and the triangles represent the 18 IGS stations. The CORS sites provide good spatial coverage within the conterminous United States (CONUS), while the IGS sites have only a fair spatial distribution. Both networks were considered to compare the data quality and the effect of multipath. Previous analysis has indicated that many of the CORS sites suffer from more multipath than the IGS. Arrow symbols indicate stations that we later removed from the solution for validation purposes.

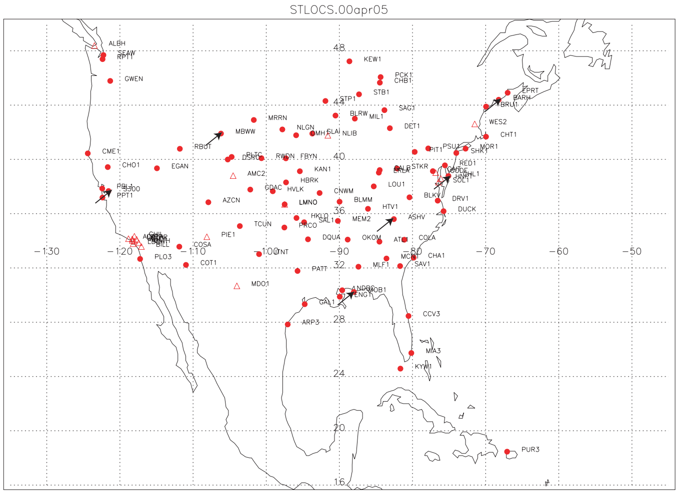

Figure 2. Network of CORS and IGS stations processed for April 5, 2000.

ANALYSIS OF RESULTS

Satellite and Receiver Instrumental Biases. We first looked at the satellite and receiver instrumental biases estimated with the old single-shell and the new multi-shell approaches. We believe that the biases are constant over a period of several days and so any variation in the day-to-day bias estimates represents un-modeled systematic errors propagating into the bias solution. Other studies such as that by Mannucci et al. [1999] also suggest that multi-layer models can reduce systematic errors in bias estimates.

We compared the bias scatter (std. dev.) using global GIM fits during the period of December 27, 2001 to January 2, 2002. Figure 3 shows that the multi-shell approach improved the standard deviation of the satellite biases over the 7-day period by a factor of 2 to 4, from 2–6 cm to 8–24 mm.

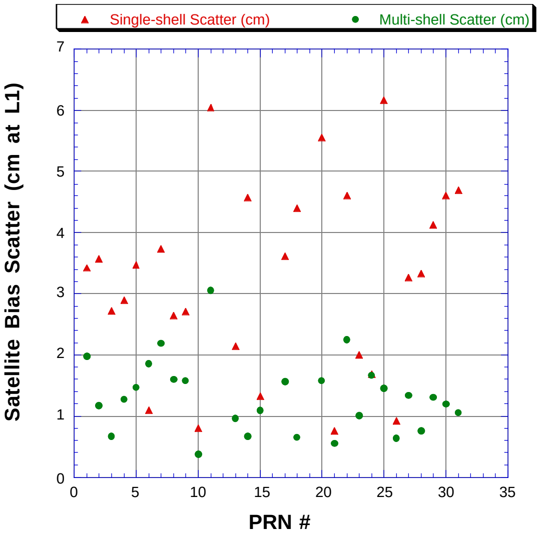

Figure 3. Comparison of satellite instrumental bias estimates.

A similar trend can be seen in the receiver bias estimates shown in Figure 4. Using the multi-shell approach, the 7-day scatter improved from 8–64 cm to 0.5–19 cm. In Figure 4, the larger scatter values are for stations in the equatorial region. Since we estimate the sum of the line-of-sight ionospheric delays and the instrumental biases (see Eqns. 1 and 2), systematic un-modeled ionospheric effects may have propagated into the single-layer bias estimates. As shown above, these systematic errors have been greatly reduced using our new multi-shell approach.

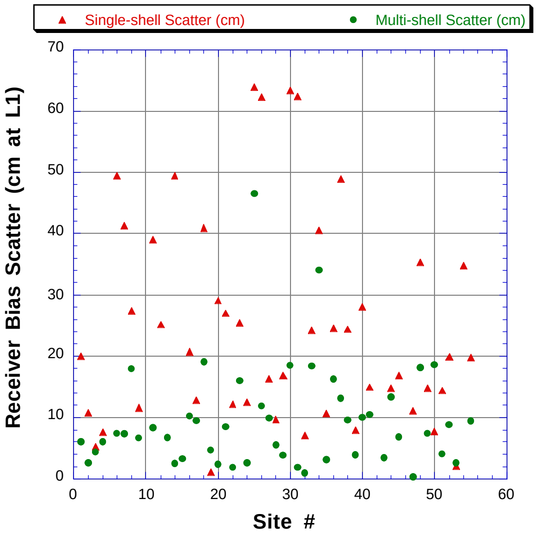

Figure 4. Comparison of receiver instrumental bias estimates.

Postfit Residuals. Next we compared the line-of-sight postfit ionospheric residuals using the single-shell and multi-shell approaches. Figure 5 shows results from both approaches for a subset of CORS stations on April 5, 2000. The multi-shell approach improves the residuals for all stations. The RMS of the L1 slant ionospheric residuals is always smaller than 0.7 meter.

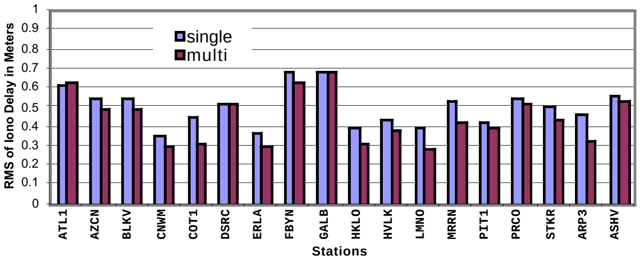

Figure 5. RMS of postfit residuals for CORS.

We also processed separately the available IGS stations for 5 April 2000. In Figure 6, we plot the RMS of the postfit slant ionospheric delay residuals. The RMS values are similar to or smaller than those for the CORS stations. However, it is interesting to see that the multi-shell improvement over single-shell is more pronounced in the case of the IGS stations, most likely due to the fact that the IGS stations are less affected by multipath than the CORS sites and so the improved ionospheric modeling is more apparent.

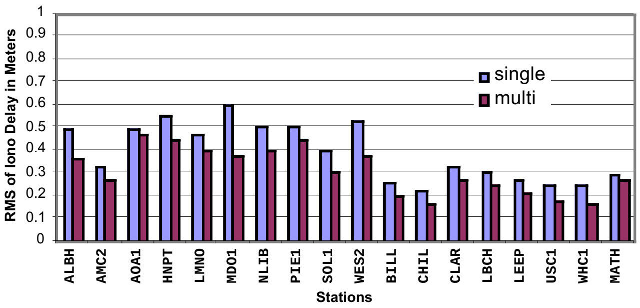

Figure 6. RMS of postfit ionospheric residuals for IGS.

Prediction Residuals. To validate the new multi-shell approach, we removed six evenly-distributed CORS stations from the network of 90 receivers within the CONUS (see Figure 2), computed a solution using the remaining 84 stations, and then formed the differences between the predicted line-of-sight ionospheric delays and the actual ones measured at the test sites. The inter-frequency receiver biases for the missing sites were estimated in a separate run that included all sites.

In Figure 7, we plot the RMS of the postfit residuals and the RMS of the prediction residuals for the six selected sites. It is seen that the multi-shell approach does better than the single-shell for all sites in prediction mode. The RMS of the prediction residuals are larger than the corresponding RMS of the postfit residuals by about 0.1 meter.

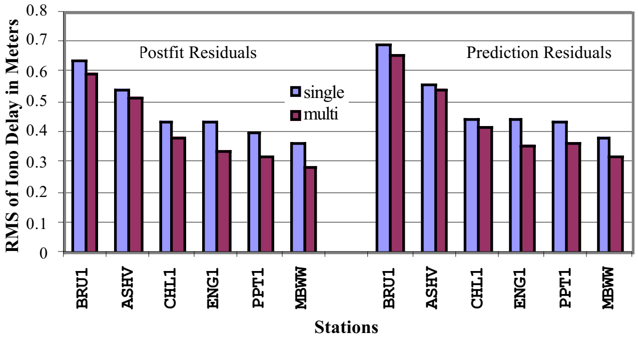

Figure 7. Comparison of postfit and prediction residuals.

Storm Data. The data set we have discussed thus far is that of 5 April 2000 and is characterized by quiet ionospheric conditions. We also processed the subsequent day during which a major geomagnetic event occurred with an Ap of 236. The RMS of postfit residuals for five stations are summarized in Figure 8, covering both the quiet and storm day conditions (station MBWW was unavailable for April 6, 2000). For 4 of the 5 stations investigated, the RMS of postfit residuals increased for the storm day compared to the quiet day, by 0.1–0.2 meter. It is evident that multi-shell performed well compared to the single-shell approach, even for the storm day.

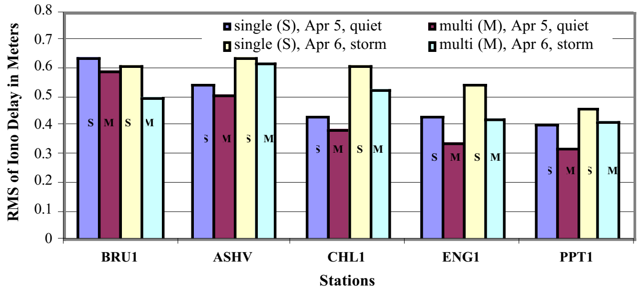

Figure 8. Comparison of postfit residuals between quiet and storm days.

Ionosphere-Induced Positioning Errors. We have investigated both the CORS and IGS sites from the CONUS sector for both quiet and storm days. Based on this limited data set, we found that we were able to achieve a better than 0.7 meter RMS of ionospheric slant path residuals even in prediction mode.

User positioning accuracy is defined as the product of PDOP and the measurement accuracy. Assuming a slant ionospheric measurement error of 0.7 meter and a PDOP value typically less than 3, we conclude that the user positioning error caused by the ionosphere is less than 2.1 meters (horizontal distance RMS).

Elevation Angle Dependency. As an example, in Figure 9 we plot the line-of-sight ionospheric delay at station ENG1 (English Turn, LA) as a function of elevation angle. The ionospheric delay ranges between 5 and 45 meters which is typical for conditions near the peak of a solar cycle such as the year 2000.

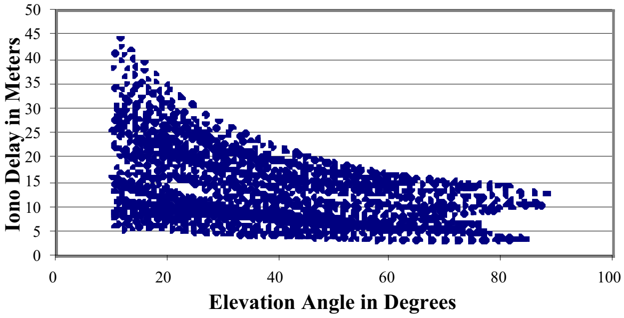

Figure 9. Slant ionospheric delay at station ENG1.

In Figure 10, we plot the ionospheric prediction residuals as a function of elevation angle. The multi-shell approach reduced the overall RMS of residuals from 0.46 to 0.37 meter. In Figure 10, with solid lines, we also plot the RMS of the ionospheric residuals for each 10-degree elevation angle bin (e.g., data in 10–20, 20–30 degree bins, etc.). It is evident that significant improvement was achieved over the single shell approach for the low elevation angle regime, e.g., an improvement of 0.2 meter RMS was achieved for elevation angles from 10 to 20 degrees. This in turn will provide improved WADGPS positioning accuracy when using data from low elevation angle satellites.

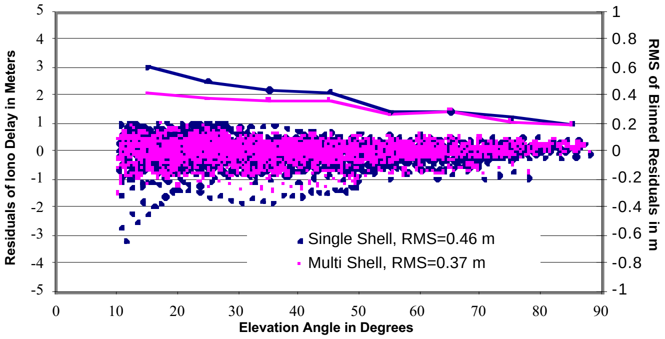

Figure 10. Ionospheric prediction residuals for station ENG1.

Figure 11 shows another example, for station ASHV (Ashville, NC). The RMS of the prediction residuals is reduced from 0.61 to 0.54 meter using the multi-shell approach. However, notice a bi-modal residual behavior when the residuals are plotted against Sun-fixed longitude in Figure 12.

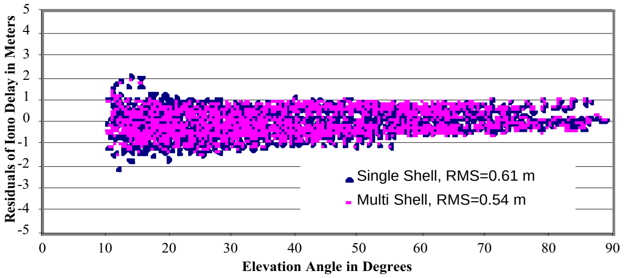

Figure 11. Ionospheric prediction residuals for station ASHV.

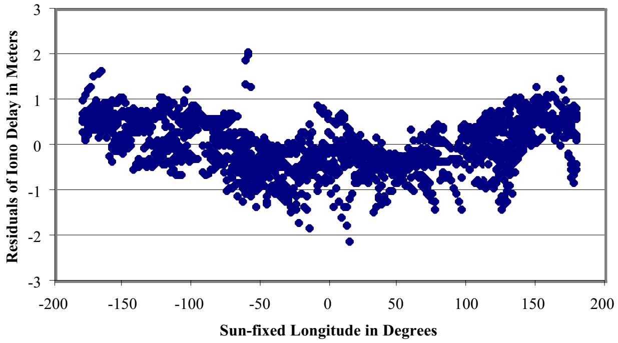

Figure 12. Ionospheric prediction residuals as a function of Sun-fixed longitude for station ASHV (0 degree longitude corresponds to local noon).

This clearly shows a systematic error with diurnal dependence and indicates that there is still room to improve our ionosphere modeling technique by exploring parameter space further.

CONCLUSIONS AND FUTURE RESEARCH

In this paper, we introduced a new ionospheric model for WADGPS applications, by extending the currently-available single-shell models. We no longer assume that the ionosphere can be approximated with a single centroid height, but instead assume that the ionosphere is made up of three separate shells that account for the time varying electron density distribution with height. Compared to a single-shell, this new multi-shell ionospheric model: (1) significantly reduces the day-to-day scatter in the bias estimates, (2) reduces the RMS of postfit residuals for all stations, (3) reduces residuals at low elevation angles as much as 0.2 meter RMS of ionospheric delay, and (4) improves prediction accuracy in the mid-latitude sector. Better slant TEC prediction accuracy enables improved user positioning accuracy using GPS data from satellites at low elevation angles.

For the April 5−6, 2000 data set, the RMS of slant ionospheric delay residuals at the L1 frequency was better than 0.7 meter for both quiet and storm days. Assuming a pessimistic PDOP number of 3, this corresponds to as much as 2.1 meter positioning error (horizontal distance RMS) caused by the ionosphere. We also found that the IGS sites are less affected by multipath than the CORS sites and so multi-shell showed more improvement over single-shell for the IGS sites.

We have positioning test comparisons between the single-shell and multi-shell approaches under way. We plan on exploring the parameter space further to improve predictions including for example the use of more than three shells. We will also investigate the possibility of using varying ionospheric shells adapted to different ionospheric latitude sectors and structures. Furthermore, examining longer time series of multi-shell bias estimates will help us better understand how systematic errors can be further minimized.

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