vector) as:

$$\frac{\mathrm{d}\ddot{\mathbf{a}}}{\mathrm{d}\mathbf{X}} = -\frac{\partial f}{\partial \mathbf{X}} rRHS - f\frac{\partial rRHS}{\partial \mathbf{X}} \tag{2.15}$$

The panel independent partials (the first term of the right-hand side in Eq. (2.15)) are computed once for reasons of computational efficiency in
`SpacecraftComponents::computeRadiationPressureAcceleration()`,
while panel dependent partial terms (second term of the right-hand side in Eq. (2.15)) are accumulated:

$$\begin{aligned} f\frac{\partial rRHS}{\partial \mathbf{X}} = fA\Bigg\{ &(1-\rho)\left[\cos(\theta)\frac{\partial \hat{\mathbf{s}}}{\partial \mathbf{X}} + \hat{\mathbf{s}}\frac{\partial \cos(\theta)}{\partial \mathbf{X}}\right] + \\ &2\cos(\theta)\frac{\partial \hat{\mathbf{n}}}{\partial \mathbf{X}}\left[\frac{\delta}{3} + \rho\,\cos(\theta)\right] + \\ &2\hat{\mathbf{n}}\frac{\partial \cos(\theta)}{\partial \mathbf{X}}\left[\frac{\delta}{3} + 2\rho\,\cos(\theta)\right]\Bigg\} \end{aligned}$$

This model is implemented in:

- `SpacecraftComponents::flatPlateRadiationPressureAcceleration_()`
- `SpacecraftComponents::computeRadiationPressureAcceleration()`
- `SpacecraftComponents.h`
- `SpacecraftComponents.cpp`

2.2.5.3.2 FSPM Fourier Solar Radiation Pressure Model

OI accepts Fourier series input via a generic customizable interface for different satellites. This is an expanded version of the JPL empirical SRP model for GPS satellites developed by Bar-Sever and Kuang [2005] in that different scale factors are allowed along each orthogonal direction of the specified coordinate system. An explicit example of such a model is GSPM.IIA.04, developed by Bar-Sever and Kuang [2005] for GPS block IIA satellites.

Eq. (2.16) represents the Fourier expansion of the solar radiation pressure force. F^j, where $j = \{0, 1, 2\}$, provides the force along each of the satellite's body-fixed axes, respectively $[x, y, z]$. The first and second summations allow the sine and cosine coefficients (S^j and C^j) to have independent sets of harmonics (taken from R_1^j and R_2^j) for their common angular argument ϕ (typically the Earth-probe-Sun angle),

$$F^j\left(C^j, S^j, R_1^j, R_2^j, \phi\right) = \sum_{m\in\left\{R_1^j\right\}} S_m^j \sin(m\phi) + \sum_{m\in\left\{R_2^j\right\}} C_m^j \cos(m\phi) \tag{2.16}$$

The satellite body-fixed accelerations due to SRP can then be expressed by Eq. (2.17),

$$a^j = \upsilon\left\{\frac{\kappa p^j\left[F^j\left(C^j, S^j, R_1^j, R_2^j, \phi\right) + pF^j\left(\widetilde{C}^j, \widetilde{S}^j, \widetilde{R}_1^j, \widetilde{R}_2^j, \phi\right)\right]}{M} + \delta_1^j C_0^j\right\} \tag{2.17}$$

where a shadow factor (υ, nominally set to 1) is available to accommodate eclipse seasons. The Kronecker delta (δ_1^j) is used to isolate C_0^1 (nominally set to 0), a coefficient typically used with GPS satellites to represent the so-called Y-bias acceleration (an anomalous acceleration in the body-fixed Y-axis direction, Fliegel et al. [1992]). Dimensionless scalar terms (nominally set to 1) may be used to apply a scale factor along each axis (p^j), and/or an axis-independent scale factor (p) to a sub-set of coefficients ($\widetilde{C}^j, \widetilde{S}^j$). The mass ($M$) of the spacecraft in kg converts force to acceleration. The term κ is an inverse distance scalar computed to adjust the nominal force, evaluated at one astronomical unit [Petit and Luzum, 2010, Table 1.1], for the current distance between the spacecraft and the Sun ($|\mathbf{r}_{\mathrm{sc}\odot}|$),

$$\kappa = \left(\frac{au}{|\mathbf{r}_{\mathrm{sc}\odot}|}\right)^2$$

A single set of coefficients is often enough to characterize the SRP model for a satellite during standard operations. However, the implementation of this model is flexible enough to allow for some variation during eclipse seasons and also during those times when the satellite is unable, for whatever reason, to follow its nominal attitude model.

Eclipse Seasons: The satellite's beta angle (β) is defined as the angle between the Earth-Sun vector and the satellite's orbital plane,

$$\beta = \frac{\pi}{2} - \cos^{-1}\left(\frac{(\mathbf{r}\times\dot{\mathbf{r}})\cdot\hat{\mathbf{r}}_\odot}{|\mathbf{r}\times\dot{\mathbf{r}}|}\right)$$

where $\mathbf{r}$ and $\dot{\mathbf{r}}$ are the position and velocity vectors of the satellite in inertial coordinates, and $\hat{\mathbf{r}}_\odot$ is the unit position vector of the Sun in inertial coordinates. Note that the right hand rule relative to the motion of the satellite is used to define the sign of β. The beta angle is used by the model to demarcate eclipse seasons, typically $-14.5° < \beta < 14.5°$, for which a different set of Fourier coefficients may be defined; the standard coefficients are used during this regime if a separate set is not provided.

Non-Nominal Attitude: An alternative force model that is partially theoretical and partially empirical may be used when the attitude of the spacecraft deviates significantly from nominal. Non-nominal attitude may, for example, occur when a GPS block IIA satellite emerges from umbra or when a GPS block IIR satellite hits its maximum yaw rate near the orbit noon point at very small β angles; both of these events are described in detail in section 2.2.6. The alternative model may be used when

$$\hat{\mathbf{r}}_\odot \cdot \hat{\mathbf{n}} > \tau \tag{2.18}$$

where $\hat{\mathbf{n}}$ is the unit normal of the solar panel array and τ is nominally set to $\cos(1°)$. If the criterion in Eq. (2.18) is met, then the SRP accelerations are computed by treating the satellite as a very simple box-wing model comprised of just two panels (see section 2.2.5.3.1): a solar panel and a bus panel. In practice, the sum of the force exerted by these two panels is typically taken to be represented directly by the C_1^2 term of the coefficients used during eclipse season (with $\phi = 0$); input can be provided to specify what fraction of the force refers to the solar panel and what fraction to the bus panel. The acceleration due to the solar panel is decomposed into accelerations normal and shear to the solar panel surface, while the bus panel acceleration is much simpler and is treated as acting purely along $\mathbf{r}_{\mathrm{sc}\odot}$.

Because the solar panel portion involves both normal and shear forces, it makes sense to re-use code implemented for 2.2.5.3.1 when computing the forces (and partials) involved. However, this means being able to compute the nominal area of such a panel. Removing the scalar term f (which contains the mass of the spacecraft) from Eq. (2.14), leaves an equation expressing the nominal force vector ($\mathbf{F}$) on a flat panel due to SRP,

$$\mathbf{F} = -A\cos(\theta)\left[2\left(\frac{\delta}{3} + \rho\,\cos(\theta)\right)\hat{\mathbf{n}} + (1-\rho)\hat{\mathbf{s}}\right] \tag{2.19}$$

To recover the panel's nominal area (A), assume that $\theta = 0$ (i.e. $\hat{\mathbf{n}} \equiv \hat{\mathbf{s}}$), $n_x = 1$ and set F_x equal to the total force from the solar panel,

$$A = \frac{-F_x}{\frac{2}{3}\delta + \rho + 1} \tag{2.20}$$

This algorithm is implemented in the following subroutines:

- `solar__pressure.cpp`
- `FourierSolarPressureModel.cpp`

The file format description can be found here:

- `FourierSolarPressureModelFormat`